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International Journal of Multiphase Flow 33 (2007) 658–674
International Journal of Multiphase Flow
www.elsevier.com/locate/ijmulflow

Mathematical expression of pressure gradient in the flow of spherical capsules less dense than water

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Received 17 August 2005; received in revised form 27 November 2006

Abstract

This study yielded a mathematical expression to calculate the pressure gradient $(\Delta P/L)_m$ of the flow of a spherical capsule train. An experimental investigation was carried out to determine pressure drops of two-phase mixture flow of spherical ice capsules and water inside the pipelines of cooling systems. Instead of ice capsules, spherical capsules made of polypropylene material whose density (870 kg/m^3) is similar to that of ice were used in the experiments. Flow behavior of the spherical capsules, 0.08 m outer diameter, was observed in the measuring section inside plexiglass pipes, 0.1 m inner diameter (ID) and 6 m in length; pressure drops were measured on the 4 m section. The investigation was carried out in the $1.2 \times 10^4 < Re < 1.5 \times 10^5$ range and under transport concentration (C_{tr}) by 5–30%. Dimensionless numbers of the physical event were found out by conducting a dimensional analysis, so that mixture density was expressed in terms of specific gravity and in situ concentration. After arriving at certain conclusions based on the relevant experimental findings and observations, empirical and mathematical models which can be used for calculation of the pressure gradient were developed. Comparison of the mathematical model with the experimental findings revealed that pressure drop values deviated by 2.7% on average for $2.5 \times 10^4 < Re < 1.5 \times 10^5$.

Keywords: Two-phase flow; Spherical capsule train; Pipeline; Pressure drop; Mathematical model; Cooling systems

1. Introduction

Utilization of spherical ice capsules in cooling systems is still relatively new method. Since the latent heat of ice is very high, the same mass discharge provides more cooling, or the cooling process is ensured to be more economical on the grounds that less mass discharge and pipes smaller in diameter are required. Utilization of spherical ice capsules which occupy 80–90% of pipe's diameter will be an innovation in cooling technology and bring various advantages.

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doi:10.1016/j.ijmultiphaseflow.2006.11.002

- Since ice capsules are large in terms of dimension, they will be able to stay without melting within the system for a longer time. Melting will be even slower if capsules are put in a plastic casing. Such plastic casing will prevent adhesion and clustering behavior of granulated ice flow.
- Ice concentration values and ice capsule velocity values reached during the flow are much higher than concentration and bulk velocity values reached during ice–water slurry flow. Therefore, the cooling capacity of the district system will increase.
- Since pipelines used for heating purposes will be used for cooling in the summer, no new investment will be required. Pipelines installed to carry water can be used for flows of an ice and water mixture, so that water and cooling demands will be met together.
- Since the capsule flow will be a single mass movement, blocking problems arising from dispersion of particles in ice–water slurry flows will be prevented. Particles will neither disperse nor accumulate at low velocities, so that it will not be necessary to set a critic velocity limit for capsule flows.
- The spherical form of capsules prevents blocking of pipe joints and ensures local losses to be minimized.

Flow parameters and flow patterns concerning utilization of a granulated ice–water or snow–water mixture in cooling systems have been examined by various authors (Kawada et al., 1998, 1999; Snoek et al., 1993; Takahashi et al., 1991). The diameter of the ice particles was maximum 12 mm, and ice concentration did not exceed 25%. However, capsule flow can ensure a maximum ice concentration by approximately 43% when the diameter ratio ($k = d/D$) is 0.8. Kawada et al. (1999), has reported potential blocking problems in the flow of ice–water slurry. In ice–water slurry flows, the fact that ice particles tend to cluster especially at low velocities and proceed in the form of bed flows contacting the upper wall of pipes at a high rate increases the pressure gradient. Therefore it is needed to operate at bulk velocities that minimize the pressure gradient for slurry flows. The fact that particles within ice-slurry flows tend to disperse or accumulate in pipeline joints causes various blocking problems to arise and excessive pressure drops to happen. However, such problems do not arise in capsule flows. Empirical expressions obtained from studies conducted on granulated ice–water or snow–water mixtures used in cooling systems cannot be used for capsule flows, because the former's slurry flow mechanism and effective parameters are different from those of the latter. Utilization of spherical ice capsules in cooling systems is in its early stages, having different flow parameters than of ice–water slurry flow.

In designing capsule train pipelines, a general model is needed to calculate pressure gradients. A generalized mathematical model is needed to calculate pressure drops which can occur with different types of capsules or pipe geometries and energy requirements depending on the pressure drops. If the energy loss of a certain pipeline is known, it will be used as a criterion to select a proper type of pump and the number of pump stations. The effect of variables which characterize the event on pressure drops is defined by using such mathematical model, so that it will be possible to find out the most economical pipe geometry, capsule to pipe diameter ratio, or velocity of flow which minimizes pressure drops. Before creating such model, the physical mechanism of the capsule-water flow must be defined well, and physical magnitudes that characterize the event and their effect on pressure drops and velocities must be determined. Another important requirement is to verify the mathematical model by conducting experiments.

This study was conducted to develop empirical and mathematical models for the calculation of pressure gradients of a spherical capsule train whose density is close to that of ice, flowing inside horizontal and straight pipes.

2. Literature review

Numerous experimental investigations were conducted on hydraulics conveyed in capsule pipelines (Table 1). The starting point for the investigations was to find out how to convey products (or materials) along long distances in the most economical way. Investigations started in the 1960s focused on conveyed materials whose densities were higher than or the same as the carrier liquid. Until the first half of the 1970s, studies focused on characteristics of the flow behaviors of single cylindrical capsules (with flat or rounded ends) or of spherical capsules placed in circular sectioned vertical or mostly horizontal pipes. The studies were limited with dispersion of the forces affecting the capsule's surface and with measurement of velocities of capsule-carrier liquid (Ellis, 1964a,b; Ellis and Bolt, 1964; Round and Bolt, 1965). In the second half of the 1970s, studies

Table 1
Pressure gradient expressions developed for different flow conditions

Author	Year	Number of capsule	Shape of capsule	Pipe geometry	Pressure gradient expressions
Latto et al.	1973	Single	Spherical	Vertical	$\frac{\Delta P_m - \Delta P_w}{(\rho_c - \rho_w)D} = \frac{2}{3}\left(\frac{d}{D}\right)^3$
Latto et al.	1973	Single	Spherical	Inclination	$\frac{\Delta P_m - \Delta P_w}{(\rho_c - \rho_w)D\sin\theta} = \frac{2}{3}\left(\frac{d}{D}\right)^3$
Kruyer–Ellis	1974	Single	Cylindrical	Horizontal	$Re_c = (D-d)(V_b - kV_c)/(1-k^2)\upsilon$ for $Re_c \leqslant 1000$ $\left(\frac{\Delta P}{L}\right)_m = \frac{19.2(V_b - kV_c)\rho_w \upsilon}{(D-d)^2(1-k^2)}$ for $Re_c > 1000$ $\left(\frac{\Delta P}{L}\right)_m = \left[\frac{V_m - kV_b}{1-k^2}\right]^{1.75}\left[\frac{0.14\rho_w \upsilon^{0.25}}{(D-d)^{1.25}}\right]$
Latto–Lee	1978	Single	Cylindrical	Vertical	$\frac{\Delta P_m - \Delta P_w}{[(\rho_c - \rho_w)gD]} = \left(\frac{d}{D}\right)^3\left(\frac{l}{d}\right)$ (l length of cylindrical capsule)
Kroonenberg	1978	Single	Cylindrical	Horizontal	$\left(\frac{\Delta P}{L}\right)_m = \frac{\lambda_p}{D}\frac{1}{2}\rho_w V_b^2 \frac{1}{\left(1+k^2\sqrt{k\frac{\lambda_p}{\lambda_c}}\right)^2}$
Govier–Aziz	1972	Train	Spherical	Horizontal	$\left(\frac{\Delta P}{L}\right)_m = \frac{2f\rho_m V_b^2}{D}$ (f, friction coefficient–Fanning)
Ellis et al.	1975	Train	Spherical	Horizontal	$\left(\frac{\Delta P}{L}\right)_m = \left[A + B\left(\frac{\Delta P}{L}\right)_w\right]k^2$ (A, B experimental constants depending on diameters of pipe and capsule)
Chow	1979	Train	Spherical	Vertical	$\frac{\Delta P_m - \Delta P_w}{(\rho_c - \rho_w)D} = 0.79\left(\frac{d}{D}\right)^{3.572} m^{1.003}$ (m, the number of capsules in train)
Agarwal–Mishra	1998	Train	Spherical	Horizontal	$\lambda = 4(a + bRe^{-c}),\quad \lambda = 4f$ $a = 0.26(e_p/D)^{0.225} + 0.133(e_p/D)$ $b = 22(e_p/D)^{0.44},\quad c = 1.62(e_p/D)^{0.134}$ $\left(\frac{\Delta P}{L}\right)_m = \frac{2f\rho_m V_b^2}{D}$
Govier–Aziz	1972	Train	Cylindrical	Inclination	$\left(\frac{\Delta P}{L}\right)_m = \psi\left(\frac{\Delta P}{L}\right)_w + fk^2\frac{\rho_m - 1}{\rho_m}(1 - C_L)\cos\theta$ (ψ coefficient depending on flow regime, C_L buoyancy coefficient)

were directed to capsule trains. However, the number of the capsules that formed the train was limited in the studies (Ellis and Kruyer, 1974; Ellis et al., 1975) Early studies conducted to find out both the flow mechanism of a single capsule and the behavior of a capsule train had used a capsule (or a capsule train) fixed or suspended inside a pipe for liquid flow. In a study conducted on hydrodynamics of moving capsules, Vlasak (1999) used capsules in anomalous shapes (i.e. cylindrical capsules with helical grooves carved on them) and having densities higher than that of the carrier liquid, and set up a special delivery mechanism to ensure the capsule train to be continuously moving within the system.

To date no mathematical model has been developed to calculate pressure drops in the flow of a spherical capsule train whose density is less than that of the carrier liquid inside horizontal pipes. Some of the empirical expressions presented in the experimental studies conducted to date involve capsule flows inside vertical pipes (Latto et al., 1973; Latto and Lee, 1978; Chow, 1979). Flow in a vertical pipe is different from flow in a horizontal pipe. Movement of capsules in a vertical pipe represents only the carriage movement exerted by water on them. Unlike movement in a horizontal pipe, neither a rolling movement (if the capsule is spherical) nor any surface friction losses occur. Therefore, empirical expressions developed for pressure drops in the flow of capsules in a vertical pipe cannot be used for capsule flows in a horizontal pipe.

In general, a single or cylindrical capsule was selected for the studies conducted on capsules flowing in a horizontal pipe (Kruyer and Ellis, 1974). In developing a model for a single cylindrical capsule whose density was higher than that of the carrier liquid, the capsule was allowed to occupy a long portion of the pipe and the end effects were ignored (Kroonenberg, 1978). Expressions developed for pressure drops in single capsule flows or in cylindrical capsule flows do not calculate pressure drops in flows of a spherical capsule train.

It was assumed in the studies conducted on cylindrical or spherical capsule trains that the capsules proceed in a concentric position (where the capsule and pipe axes overlap) and that the capsules contact each other. Flow of the capsule and carrier liquid mixture was considered a single-phase homogenous flow, so that losses caused by the rolling movement were ignored especially in spherical capsules (Govier and Aziz, 1972; Agarwal and Mishra, 1998). In fact, the capsule movement mechanism mostly consists of rolling and rolling + sliding (flowing), therefore a general model must contain the losses caused by rolling movement.

For this study, experimental equipment similar to a capsule pipeline was installed. Spherical capsules made of polypropylene material whose density ($870\ \mathrm{kg/m^3}$) is similar to that of ice were used in the experiments. Flow behavior of the spherical capsules, 0.08 m outer diameter, was observed in the measuring section inside plexiglass pipes (ID 0.1 m). In developing a model, the flow mechanism of a spherical capsule train whose density is less than that of water was taken as a basis. Distance between the capsules, roughness of the capsule and pipe surfaces, and frictional losses caused by the rolling movement of the spherical capsules were taken into account in developing a mathematical model for pressure drops.

Previous studies conducted by this author had developed methods to measure actual velocity of the capsules, distance between the capsules, and concentration of the capsules (Ulusarslan, 2003; Ulusarslan and Teke, 2005). This paper contains a brief summary of the previous studies and a dimensional analysis, and explains the conclusions arrived with reference to the relevant findings and how to determine the density of the mixture. This paper also develops empirical and mathematical expressions based on experimental findings, and explains the deviations between the experimental findings and models.

3. Dimensional analysis

Buckingham's π method was used for dimensional analysis. Independent variable parameters of the system are capsule velocity (V_c), capsule diameter (d), capsule density (ρ_c), the mean distance between capsules (l_c), pipe diameter (D), the density of carrier liquid (ρ_w), dynamic viscosity of carrier liquid (μ_w), and the average velocity of capsule and carrier liquid flowing together (bulk velocity) (V_b). L refers to the length of the pipe in the measuring line; the pressure drop of the mixture per unit length (pressure gradient) depends on the independent variables of the system.

$$V_c \text{ or } \left(\frac{\Delta P}{L}\right)_m = f(V_b, D, d, \rho_c, \rho_w, \mu_w, l_c, g) \tag{1}$$

seven dimensionless numbers are obtained from the physical event expressed in terms of three basic dimensions (*mass, length, time*) and composed of eight independent variables. The velocity ratio ($R_v = V_c/V_b$) or the dimensionless pressure gradient can be written with reference to the literature (Ellis, 1964a; Round and Bolt, 1965; Vlasak, 1999) as a function of those five dimensionless numbers.

$$\frac{V_c}{V_b} = f\left(\frac{d}{D}, \frac{\rho_c}{\rho_w}, \frac{V_b\rho_w D}{\mu_w}, \frac{V_b^2}{gd}, \frac{d}{l_c}\right) \tag{2}$$

$$\frac{V_b^2\rho_w}{D}\left(\frac{\Delta P}{L}\right)_m^{-1} = f\left(\frac{d}{D}, \frac{\rho_c}{\rho_w}, \frac{V_b\rho_w D}{\mu_w}, \frac{V_b^2}{gd}, \frac{d}{l_c}\right) \tag{3}$$

In accordance with the results of the dimensional analysis, the dimensionless numbers of the system were written as an expression of the dimensionless pressure gradient $\left(\frac{V_b^2\rho_w}{D}\left(\frac{\Delta P}{L}\right)_m^{-1}\right)$ or as a function of the velocity ratio ($R_v = V_c/V_b$). Dimensionless numbers are diameter ratio ($k = d/D$), the specific gravity of the capsule ($s = \rho_c/\rho_w$), *Re* number ($Re = V_b\,\rho_w\,D/\mu_w$), the Froude number of the capsule ($Fr = V_b^2/gd$) and the expression that yields in situ concentration (d/l_c) (Fig. 1), which describes the part of the pipe occupied by the capsule. In situ concentration is relative linear filling of the capsules in the train, but only in longitudinal

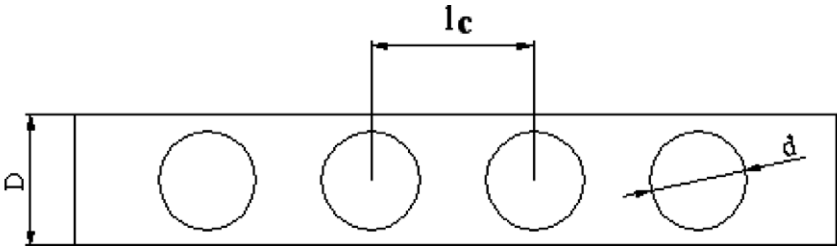


Fig. 1. Schematic diagram of distance between capsules.

dimension. The dimensionless number of (d/l_c) expresses the in situ concentration. The volumetric in situ concentration was found out to be:

$$C_{\text{in situ}} = \frac{2}{3}k^2\frac{d}{l_c} \tag{4}$$

Diameter ratio ($k = 0.8$) and the specific gravity of the capsule ($s = 0.87$) are constant in the experiments. Relative surface roughness of pipe and capsules are zero ($e_c/d = 0$, $e_p/D = 0$). Investigations carried out by using spherical capsules in equal density reported that the influence of the Froude number is negligible (Ellis and Bolt, 1964). The density of the spheres used in this investigation's system was near to that of water. Therefore, the Froude number was not taken into account for calculations. In this case, the velocity ratio and the dimensionless pressure gradient are expressed as a function of the *Re* number and the capsule concentration.

$$C_{\text{tr}} = \frac{C_{\text{in situ}}V_c}{V_b} \tag{5}$$

$$\frac{V_c}{V_b} = f\left(\frac{V_b\rho_w D}{\mu_w}, \frac{d}{l_c}\right) \quad \text{or} \quad \frac{V_c}{V_b} = f\left(\frac{V_b\rho_w D}{\mu_w}, C_{\text{tr}}\right) \tag{6}$$

$$\left(\frac{V_b^2\rho_w}{D}\left(\frac{\Delta P}{L}\right)_m^{-1}\right) = f\left(\frac{V_b\rho_w D}{\mu_w}, \frac{d}{l_c}\right) \quad \text{or} \quad \left(\frac{V_b^2\rho_w}{D}\left(\frac{\Delta P}{L}\right)_m^{-1}\right) = f\left(\frac{V_b\rho_w D}{\mu_w}, C_{\text{tr}}\right) \tag{7}$$

4. Experimental equipment and procedures

An experimental set-up, similar to actual capsule pipelines, was created (Ulusarslan, 2003). A schematic diagram of the experimental set-up is shown in Fig. 2.

Water is pumped from the tank to the system by a scroll type of pump having a horizontal shaft and a power output of 7.5 kW. The flow rate of the water in the system was regulated through a valve connected

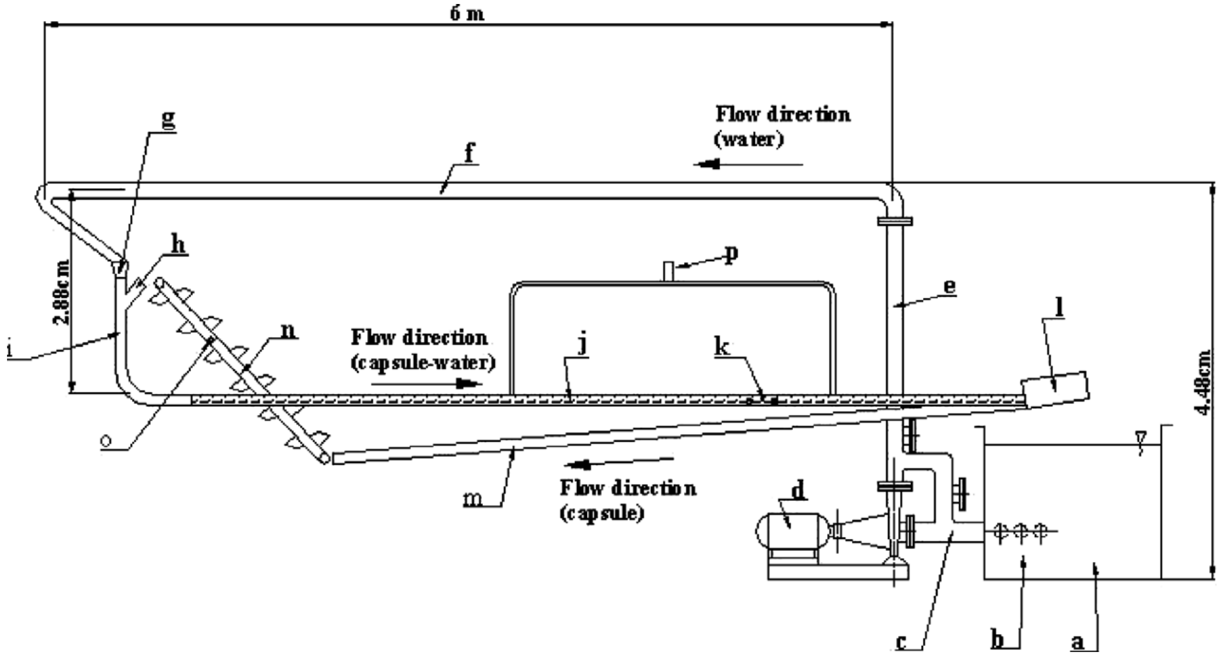


Fig. 2. Schematic diagram of the experimental set-up; a – Tank, b – Flow meters, c – By-pass line, d – Pump, e – Pressurized pipe, f – PVC pipe, g – Reverse reduction, h – Y branch, I – PVC pipe, j – Plexiglass pipe, k – Sensors, l – Sieve, m – Capsule feeding pipe, n – Elevator, o – Sensor, p – Transmitter.

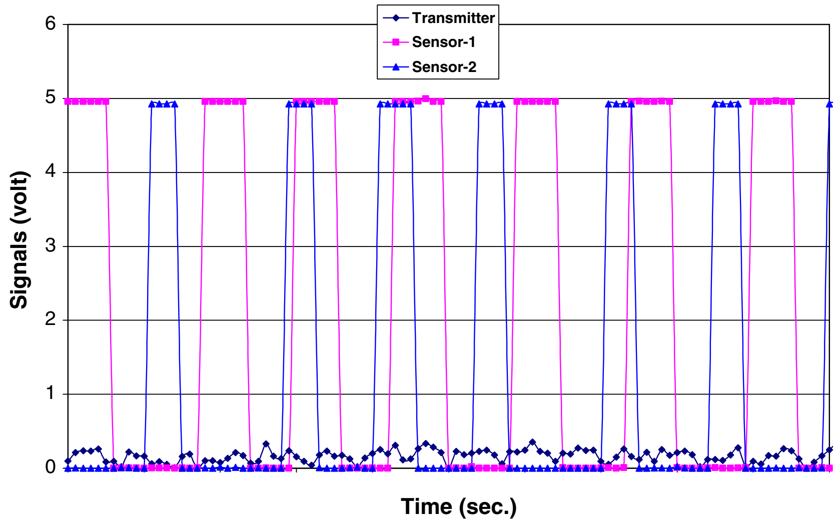


Fig. 3. Graphic of the signals transferred from the transmitter and the sensors to the computer.

to the pump outlet and the amount of flow rate was measured by water flow meters having an average variation of 0.5% (max. 1%) from the actual values.

The capsules were fed into the system through a 3 m. long, 16-bucket elevator. The capsule concentration in the system was controlled by regulating the elevator speed with a driver. The number of capsules (N) released into the system per unit time was determined by considering the driver speed (rpm) with the help of a fiber optic sensor located on the elevator.

The test section was formed by means of horizontal plexiglass pipes 6 m long, 0.1 m inner diameter. Pressure drop measurements were carried out on the 4 m section of the plexiglass pipe. Two pressure taps were connected through piezometric hoses to the ends of a differential pressure transmitter used for measuring pressure drops with a distance (L) of 4 m between them. The transmitter was able to measure pressure changes at a range of 0–10 kPa with an accuracy of 0.5%.

Two fiber optic sensors were installed on the measuring section with a distance of 0.1 m between them. Signals transmitted from the printed circuit board to the computer were determined by means of special software to calculate capsule velocities, the number of capsules that would constitute in situ and transport concentrations, and the distance between the capsules.

Analog output (0–10 V) coming from the transmitter and from the sensors of the measuring section were converted into digital data through an industrial automation printed circuit board installed in the computer and an automation program set for the test strategy. A graphic of the original signals obtained from the transmitter and the sensors is shown in Fig. 3.

Capsules passing through the measuring section were re-fed into the system through a sieve installed on the tank to ensure the continuous flow of the capsule train. Capsules comprising the train were rigid, in spherical shape, with a specific gravity of 0.87 and a diameter ratio of 0.8. The temperature of the water circulating within the system during the tests was $20 \pm 2\,^{\circ}\mathrm{C}$. Pressure drops were measured at the $1.2 \times 10^4 < Re < 1.5 \times 10^5$ range and under transport concentrations of 5–30%.

5. Experimental parameters

5.1. Capsule concentration

Volumetric capsule concentration can be determined in terms of transport concentration and in situ concentration (Ulusarslan, 2003; Ulusarslan and Teke, 2005, 2006). Transport concentration is defined as the ratio of the volumetric flow rate of the solid materials moving in the system to the total flow rate.

$$C_{\mathrm{tr}} = \frac{Q_{\mathrm{c}}}{Q_{\mathrm{w}} + Q_{\mathrm{c}}} = \frac{\left(\frac{\pi d^3}{6}\right)N}{Q_{\mathrm{w}} + \left(\frac{\pi d^3}{6}\right)N} \tag{8}$$

Volumetric flow rate of the water measured by the water meters is Q_{w}; volumetric flow rate of the capsules in the pipeline is Q_{c}. The number of the capsules fed to the pipeline per unit time is N, as calculated by evaluating the sensors' signals. The change of N determines transport concentration at different values of the volumetric water flow rate.

In situ concentration refers to the ratio of the capsule volume to the total volume in the pipe, which was 1 m long. Results of the test indicate that the flow velocities of the phases are very close to each other.

$$C_{\mathrm{tr}} = C_{\mathrm{in\ situ}} R_{\mathrm{v}} \tag{9}$$

Therefore, in situ concentration was considered to be equal to transport concentration; C_{tr} was used instead of $C_{\mathrm{in\ situ}}$ in the graphics.

5.2. Bulk velocity

The mean velocity reached while the capsules and water flow together yields bulk velocity, which was expressed as follows. A shown in Eq. (10) refers to the cross section of the pipe in the measuring section.

$$V_{\mathrm{b}} = \frac{Q_{\mathrm{w}} + Q_{\mathrm{c}}}{A} \tag{10}$$

5.3. Velocity ratio (R_v)

When the ratio of capsule velocity to bulk velocity is defined as the velocity ratio:

$$R_{\mathrm{v}} = \frac{V_{\mathrm{c}}}{V_{\mathrm{b}}} \tag{11}$$

V_{c}, is the actual velocity of the capsule flowing with water, found by evaluating the sensor signals transmitted to the computer.

5.4. Pressure gradient ratio (R_p)

It is defined as the ratio of pressure gradient occurring in the two-phase mixture flow (capsule train and water) to the pressure gradient occurring in the single-phase water flow.

$$R_{\mathrm{p}} = \frac{(\Delta P/L)_{\mathrm{m}}}{(\Delta P/L)_{\mathrm{w}}} \tag{12}$$

5.5. Density of mixture

The increase of $(\Delta P)_{\mathrm{m}}$ changes in proportion with square of velocity and with density. The density of the mixture was calculated by dividing the total mass of the pipe in length L by the total volume of the same, and the product was used as the density value. Where the number of capsules in a pipe in length L (1 m) is $N_{\mathrm{in\ situ}}$, the density of the mixture can be determined as

$$\rho_{\mathrm{m}} = \frac{\left(\frac{\pi D^2}{4}L - \frac{\pi d^3}{6}N_{\mathrm{in\ situ}}\right)\rho_{\mathrm{w}} + \frac{\pi d^3}{6}N_{\mathrm{in\ situ}}\rho_{\mathrm{c}}}{\frac{\pi D^2}{4}L} \tag{13}$$

Since $N_{\mathrm{in\ situ}} = L/l_{\mathrm{c}}$ and $s = \rho_{\mathrm{c}}/\rho_{\mathrm{w}}$, Eq. (13) can be simplified to determine the density of the mixture as follows:

$$\rho_{\mathrm{m}} = \frac{\rho_{\mathrm{w}} \frac{L}{l_{\mathrm{c}}} \left(\frac{\pi D^2}{4} l_{\mathrm{c}} - \frac{\pi d^3}{6} + \frac{\pi d^3}{6} \frac{\rho_{\mathrm{c}}}{\rho_{\mathrm{w}}} \right)}{\frac{\pi D^2}{4} L} \tag{14}$$

$$\rho_{\mathrm{m}} = \rho_{\mathrm{w}} \left[1 + \frac{2}{3} k^2 \frac{d}{l_{\mathrm{c}}} (s-1) \right] \tag{15}$$

$l_{\mathrm{c}} = d$ is the limit value of this study. When assuming that the capsules contact each other, $l_{\mathrm{c}} = d$. Eq. (15) is arranged as follows:

$$\rho_{\mathrm{m}\infty} = \rho_{\mathrm{w}} \left[1 + \frac{2}{3} k^2 (s-1) \right] \tag{16}$$

If the density of the mixture is expressed for in situ concentration:

$$C_{\text{in situ}} = \frac{\frac{\pi d^3}{6} N_{\text{in situ}}}{\frac{\pi D^2}{4} L} \tag{17}$$

$$C_{\text{in situ}} = \frac{2}{3} k^2 \frac{d}{l_{\mathrm{c}}} \tag{18}$$

where $C_{\text{in situ}}$ is expressed as follows when it is added to Eq. (15):

$$\rho_{\mathrm{m}} = \rho_{\mathrm{w}} [1 + C_{\text{in situ}} (s-1)] \tag{19}$$

Average distances (l_{c}) between the capsules were measured by using the sensors installed on the experimental equipment and making a photographic investigation (Fig. 4) (see Table 2) (Ulusarslan and Teke, 2005). The l_{c}

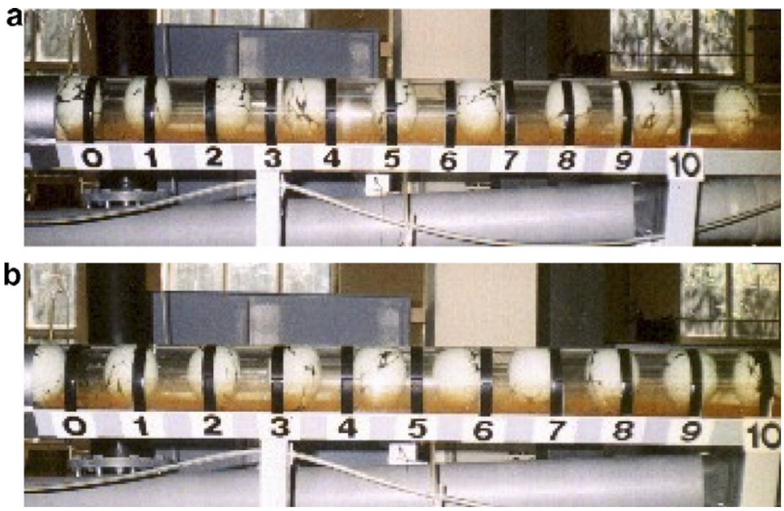


Fig. 4. Flow photograph: (a) $C_{\mathrm{tr}} = 25\%$, $N_{\text{in situ}} = 7.022$ capsules and (b) $C_{\mathrm{tr}} = 30\%$, $N_{\text{in situ}} = 8.349$ capsules.

Table 2
Densities of mixture calculated by means of Eq. (15)

$C_{\text{in situ}}$	l_{c} (m) (experimental average value of capsule distance)	ρ_{m} (kg/m^3)	$C_{\text{in situ}}$	l_{c} (m) (experimental average value of capsule distance)	ρ_{m} (kg/m^3)
0.05	0.77	994.24	0.20	0.186	976.14
0.10	0.378	988.26	0.25	0.142	968.75
0.15	0.257	982.73	0.30	0.12	963.02

values were added to Eq. (15) to calculate the mixture's density individually for each concentration. Thus mixture densities depending on experimental findings were calculated.

6. Capsule flow mechanism

Since the pipe was horizontal and the density of the capsules was lower than that of the water used as carrier liquid, the capsules proceeded with a rolling motion on the upper surface of the pipe due to the buoyancy of the water. As the mixture's velocity increased, so did the velocity of the capsules; the capsules were observed to slightly jump towards the axis. The reason of the jumps is that as the velocity of the mixture increases, the rolling velocity of the spherical capsules increases too, causing the turbulence strength of the water surrounding the spherical capsules to increase, and causing the spherical capsules to move away from the upper surface of the pipe (Magnus Effect) (Ellis, 1964b). This mechanism causes frictional forces to decrease and the spherical capsules to tend to slip. In high mixture velocities and capsule concentrations, the spherical capsule train is expected to move away from the surface of the pipe and to proceed by sliding. In such sliding movement, the capsules will proceed around the pipe's axis where maximum flow velocities occur, with a less rolling movement. Studies on this physical mechanism are in progress.

Velocity ratio increased with increasing *Re* number. At the $1.2 \times 10^4 < Re < 1.5 \times 10^5$ range and for all concentrations, average R_v was found to be 1.05 (Fig. 5). Fig. 5 can be interpreted that at all concentration values, the more the *Re* number raises, the more the velocity ratios increase. The reason of this process is that when the capsules reach high velocities, they tend to slide towards the pipe's axis where the maximum velocity occurs. As concentration increases, the capsules slide towards the axis at lower *Re* numbers. As the distances between the capsules get shorter and as the *Re* number raises, the capsule flow resembles a homogenous flow. It was observed in this experimental set-up that when the concentration values raise at higher *Re* number, an event of jump (reaching the limit value) occurs, but the effect of this jump was not included in the graphics.

7. Empirical expression

Pressure drops occurring at different *Re* numbers and capsule concentrations were measured. After having calculated the mean of the 1050 pressure drop values transferred to the computer, pressure gradients $(\Delta P/L)_m$ of the two-phase capsule train-water mixture flow were calculated. It was observed that due to the presence of the solid phase, pressure gradient increased more at capsule flow than single phase (Fig. 6).

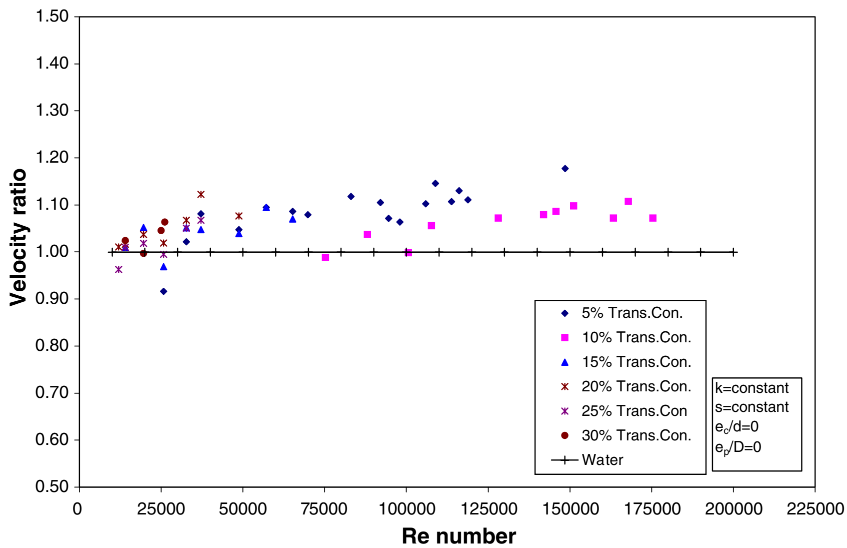


Fig. 5. The relationship between velocity ratio and *Re* number.

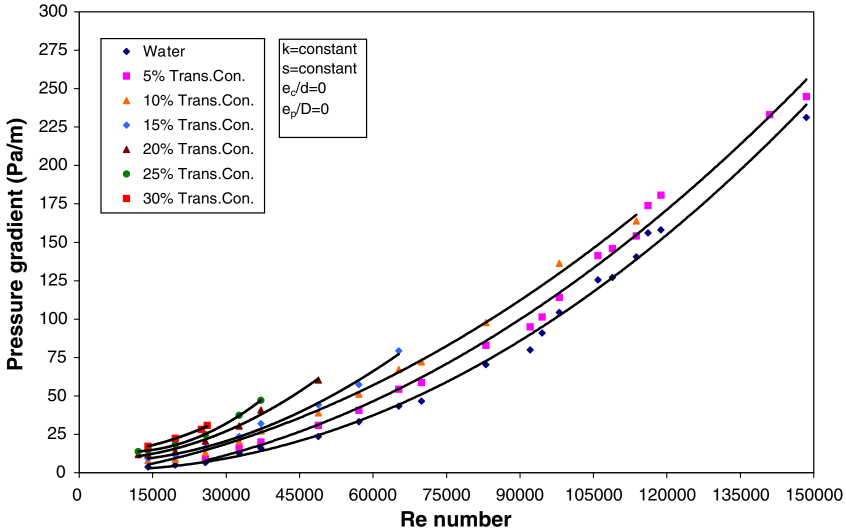


Fig. 6. Relation between $(\Delta P/L)_m$ and Re number according to experimental findings.

At fixed concentrations (fixed number of capsules), the pressure gradient ratio decreases (getting closer to 1) with increasing Re number (Fig. 7). The reason of this process is that as the flow velocity increases, the homogenous flow exerts an effect on the pressure gradient of the mixture which is higher than the rotational effect of the capsules. Losses caused by the rotation of the capsules is proportional to the flow velocity. However, the effect of the mixture flow at axial direction on pressure losses is directly proportional with the square of the flow velocity. Therefore, the more the Re number increases, the less the capsules' rotation contributes to increase of pressure losses. At lower flow velocities, the increase of pressure drops due to rotation of the capsules is more apparent.

The Darcy–Weisbach equation was taken as a basis for the expression based on experimental findings i.e. empirical expression. The Re number was calculated for velocity V_b and was used for calculating the frictional coefficient. Based on the results obtained from experiments conducted with water, the surfaces of the pipes

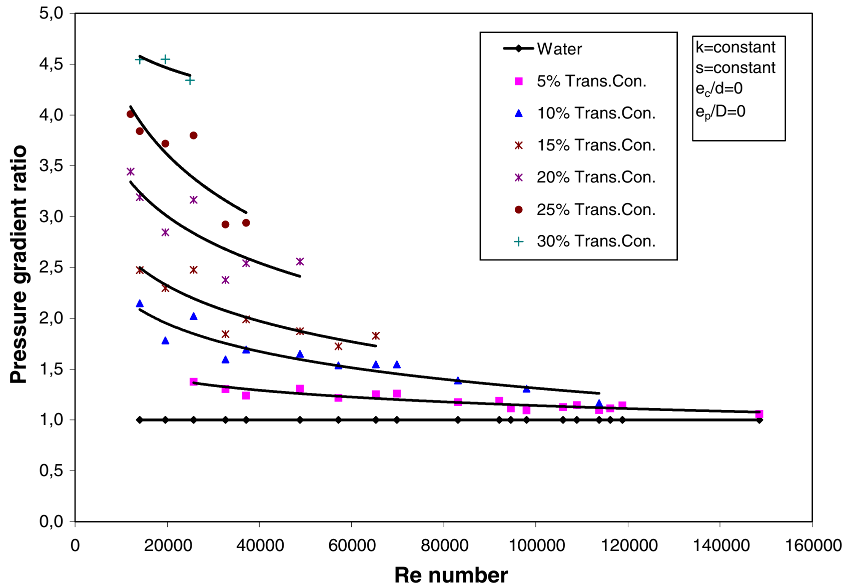


Fig. 7. Variation of pressure gradient ratio with Re number and concentration.

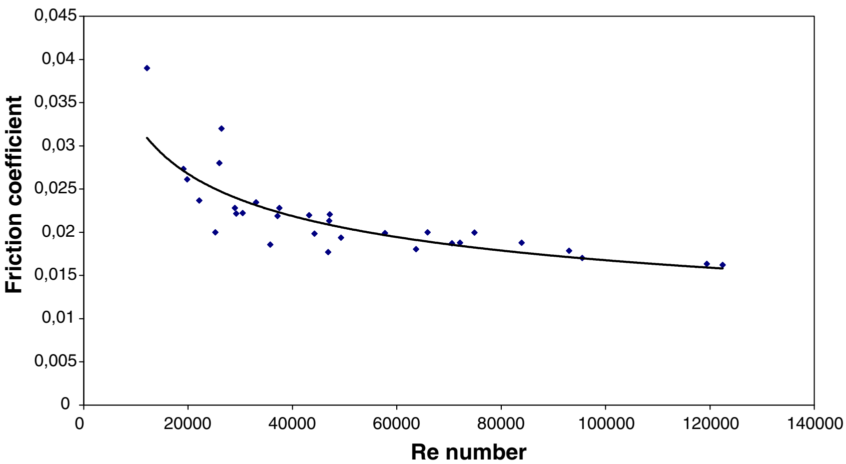


Fig. 8. Darcy–Weisbach friction coefficient versus Reynolds number for single phase water flow.

were assumed smooth in hydraulic terms (Fig. 8). For smooth pipe, the Re number depending on velocity V_b and λ values (for pipe) were selected by using the Moody diagram.

$$Re = \frac{V_b D}{\upsilon} \tag{20}$$

$$\upsilon = \mu_w/\rho_w \tag{21}$$

$$\lambda_p = f\left(Re, \frac{e_p}{D} = 0\right) \quad \text{(Moody)} \tag{22}$$

Since the surface area of the rolling spheres increases as concentration increases, it was assumed that the concentration expression influences pressure drop in the form of $(1 + C)^n$.

The effect of shear stress diminishes as the d/D rate decreases in the rolling movement of the spheres inside the pipe i.e. pressure drop decreases. The effect of diameter ratio on pressure drop was expressed as $(1 + k)^p$. Concluding that increases of d/D and of concentration are in proportion with the increase of pressure drop, the Darcy–Weisbach expression was arranged as follows for experimental findings:

$$(\Delta P)_m = \lambda_p \frac{L}{D} \rho_m \frac{V_b^2}{2} (1 + C_{\text{in situ}})^n \left(1 + \frac{d}{D}\right)^p \tag{23}$$

Forty pairs of equations with two variables were written to calculate the experimental constants n and p shown in Eq. (23). Under the conditions $2.5 \times 10^4 < Re < 1.5 \times 10^5$, it was calculated that $n = 2.88$ and $p = -0.02$.

$$(\Delta P)_m = \lambda_p \frac{L}{D} \rho_m \frac{V_b^2}{2} (1 + C_{\text{in situ}})^{2.88} \left(1 + \frac{d}{D}\right)^{-0.02} \tag{24}$$

It was observed that for $2.5 \times 10^4 < Re < 1.5 \times 10^5$, the $(\Delta P)_m$ values calculated by means of Eq. (24) (empirical expression) are compatible with the $(\Delta P)_m$ values measured experimentally with an average deviation by 3.37%.

8. Mathematical expression

Total pressure drop occurring in the flow of a mixture of capsule and water per unit length can be expressed as the total value of the pressure drop caused by the flow of the single-phase mixture (homogenous flow) and the pressure drop caused by the rolling movement of the spheres (Ulusarslan and Teke, 2006).

$$\left(\frac{\Delta P}{L}\right)_m = \left(\frac{\Delta P}{L}\right)_{\text{hom.}} + \left(\frac{\Delta P}{L}\right)_{\text{c,rolling}} \tag{25}$$

Pressure drop occurring in the flow of a single phase mixture (homogenous flow) per unit length:

$$\left(\frac{\Delta P}{L}\right)_{\text{hom.}} = \lambda_{\mathrm{p}} \frac{1}{D} \rho_{\mathrm{m}} \frac{V_{\mathrm{b}}^2}{2} \tag{26}$$

Pressure drop $(\Delta P)_{\mathrm{c,rolling}}$ caused by the rolling movement of capsules can be calculated with reference to the force balances of a flow comprised of the rolling movement of a single capsule (Fig. 9). In this case the pressure drop occurring due to the flow of a single capsule per unit length will be $(\Delta P/l_{\mathrm{c}})$.

$$\left(\frac{\Delta P}{L}\right)_{\mathrm{c,rolling}} = \frac{\Delta P}{(l_{\mathrm{c}})} \tag{27}$$

It was assumed that the resultant force is applied from the center of the sphere and creates a rolling moment against the contact point O. This moment will be balanced with a shear stress occurring at the opposite direction on the surface of a cylinder whose length equals to its diameter d. Considering the shear stress around the sphere (τ_{c}) to be a constant, the moment balance in question can be expressed as follows:

$$\Delta P \frac{\pi d^2}{4} \frac{d}{2} = \tau_{\mathrm{c}} 4\pi \left(\frac{d}{2}\right)^2 \frac{d}{2} \tag{28}$$

$$\Delta P = 4\tau_{\mathrm{c}} \tag{29}$$

Assuming that the sphere rolls along the pipe without sliding and that the center of the sphere proceeds at an average velocity, the shear stress occurring on the surface of the sphere can be calculated as follows with reference to the average space $[(D - d)/2 = \text{constant}]$ between the pipe and the sphere:

$$\tau_{\mathrm{c}} = \frac{\mu_{\mathrm{w}} V_{\mathrm{b}}}{\frac{(D-d)}{2}} \tag{30}$$

Adding Eqs. (29) and (30) into Eq. (27):

$$\left(\frac{\Delta P}{L}\right)_{\mathrm{c,rolling}} = \frac{4\tau_{\mathrm{c}}}{l_{\mathrm{c}}} = \frac{4\mu_{\mathrm{w}} V_{\mathrm{b}}}{\frac{(D-d)}{2}(l_{\mathrm{c}})} \tag{31}$$

If the units set forth in the denominator of the foregoing equation are expressed as in situ concentration (Eqs. (17) and (18)) and diameter ratios, Eq. (31) will become as follows:

$$\left(\frac{\Delta P}{L}\right)_{\mathrm{c,rolling}} = \frac{12\mu_{\mathrm{w}} V_{\mathrm{b}} C_{\text{in situ}}}{(1-k)} \frac{D}{d^3} \tag{32}$$

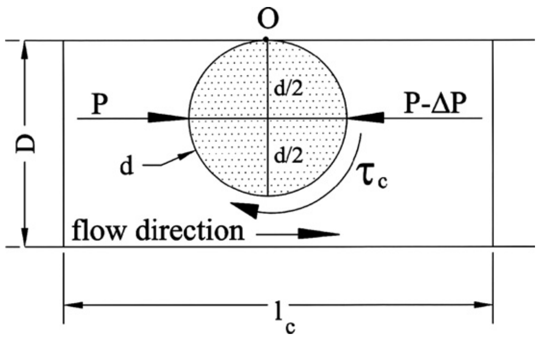


Fig. 9. Distribution of forces on spherical capsule surface according to theoretical approach.

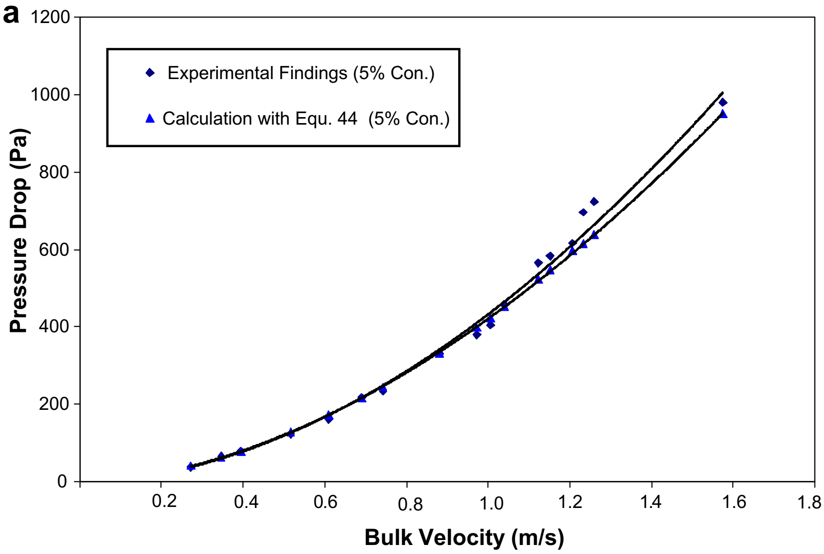
Eq. (25) can be written as follows:

$$\left(\frac{\Delta P}{L}\right)_{\rm m} = \left(\frac{\Delta P}{L}\right)_{\rm hom.}\left[1+\frac{\left(\frac{\Delta P}{L}\right)_{\rm c,rolling}}{\left(\frac{\Delta P}{L}\right)_{\rm hom.}}\right] \tag{33}$$

Adding the expression (Eq. (32)) of the pressure drop occurring in the flow of the capsule in Eq. (33):

$$\left(\frac{\Delta P}{L}\right)_{\rm m} = \lambda_{\rm p}\frac{1}{D}\rho_{\rm m}\frac{V_{\rm b}^2}{2}\left[1+\frac{\frac{12\mu_{\rm w}V_{\rm b}C_{\rm in\ situ}}{(1-k)}\frac{D}{d^3}}{\lambda_{\rm p}\frac{1}{D}\rho_{\rm m}\frac{V_{\rm b}^2}{2}}\right] \tag{34}$$

$$\frac{\left(\frac{\Delta P}{L}\right)_{\rm c,rolling}}{\left(\frac{\Delta P}{L}\right)_{\rm hom.}} = 24\frac{\mu_{\rm w}}{(1-k)k^2}\frac{C_{\rm in\ situ}}{\rho_{\rm m}}\frac{1}{d}\frac{1}{\lambda_{\rm p}}\frac{1}{V_{\rm b}} \tag{35}$$



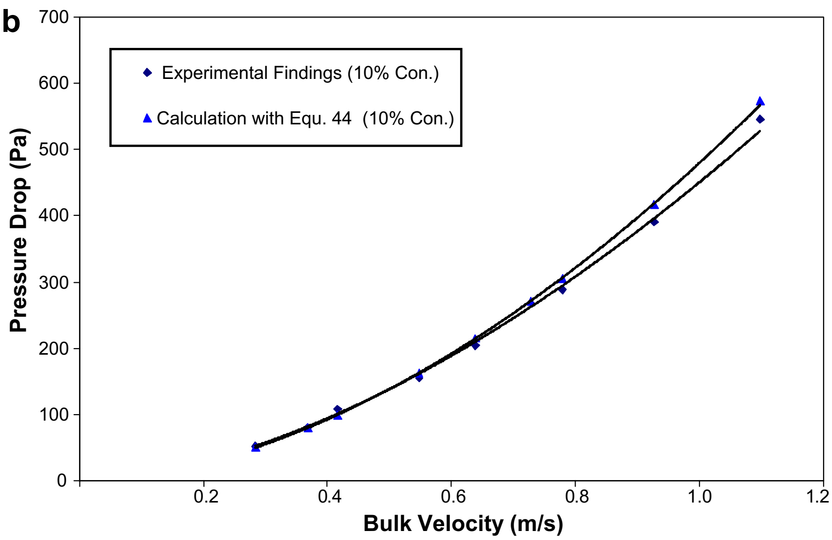
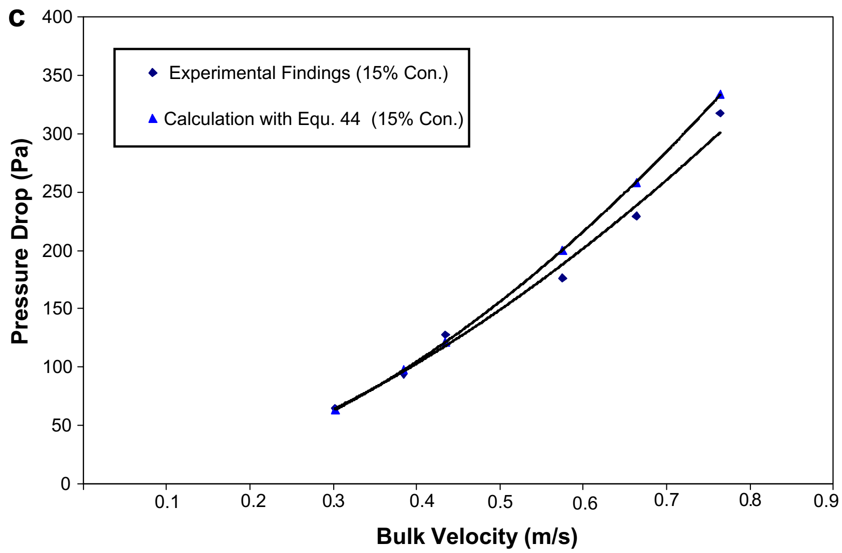


Fig. 10. Comparison of pressure drop calculated through Eq. (44) with experimental findings.



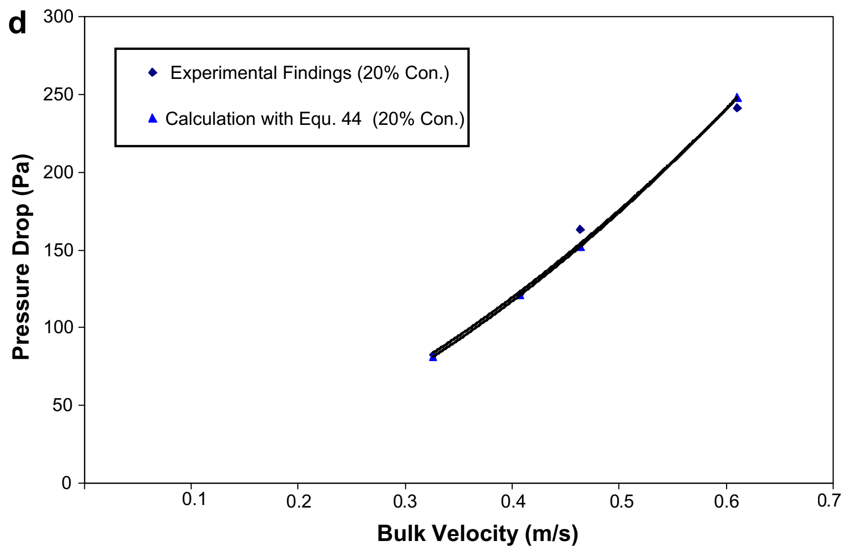


Fig. 10 (*continued*)

Eq. (34) can be simplified as necessary to calculate dimensionless numbers. Since it is as follows according to Eq. (19):

$$\frac{\rho_w}{\rho_m} = \frac{1}{1 + \frac{2}{3}\left(\frac{d}{D}\right)^2 \frac{d}{l_c}(s-1)} \tag{36}$$

Eq. (34) (mathematical expression) should be rearranged as follows.

$$\left(\frac{\Delta P}{L}\right)_m = \lambda_p \frac{1}{D} \rho_m \frac{V_b^2}{2}\left[1 + 24\frac{C_{\text{in situ}} v}{(1-k)k^2 d[1 + C_{\text{in situ}}(s-1)]\lambda_p V_b}\right] \tag{37}$$

$$l_c = \frac{2d^3}{3D^2}\frac{1}{C_{\text{in situ}}} = \frac{2}{3}k^2 \frac{d}{C_{\text{in situ}}} \tag{38}$$

A mathematical expression that characterizes the event can be written as follows by writing the expression l_c for in situ concentration and by making the necessary simplification.

$$\left(\frac{\Delta P}{L}\right)_{\mathrm{m}} = \lambda_{\mathrm{p}} \frac{1}{D} \rho_{\mathrm{m}} \frac{V_{\mathrm{b}}^2}{2} \left[1 + 24 \frac{C_{\text{in situ}} v}{(1-k)k^2[1 + C_{\text{in situ}}(s-1)]\lambda_{\mathrm{p}} d V_{\mathrm{b}}}\right] \tag{39}$$

$$\left(\frac{\Delta P}{L}\right)_{\mathrm{m}} = \lambda_{\mathrm{p}} \frac{1}{D} \rho_{\mathrm{m}} \frac{V_{\mathrm{b}}^2}{2} \left[1 + Y \frac{C_{\text{in situ}} \lambda_{\mathrm{c}}}{(1-k)k^2[1 + C_{\text{in situ}}(s-1)]\lambda_{\mathrm{p}}}\right] \tag{40}$$

$$\frac{1}{Re_{\mathrm{c}}} = \frac{v}{d V_{\mathrm{b}}} \tag{41}$$

$$\lambda_{\mathrm{c}} = f\left(Re_{\mathrm{c}}, \frac{e_{\mathrm{c}}}{D} = 0\right) \tag{42}$$

The second dimensionless term of Eq. (39) now contains the capsules' Reynolds number. The capsules' *Re* number reflects as λ_c to the numerator, and kinematic viscosity (v) was considered to be μ_w/ρ_w. λ_c represents the friction loss caused by shear stress occurring around the capsules due to rotational movement. Its value will change depending on the capsules' rotational velocity. λ_p represents the friction loss caused by shear stresses occurring on the pipe's surface at axial direction while the mixture is flowing. Calculations were made by considering $\lambda_p = \lambda_c$. Therefore Eq. (40) was used as follows.

$$\left(\frac{\Delta P}{L}\right)_{\mathrm{m}} = \lambda_{\mathrm{p}} \frac{1}{D} \rho_{\mathrm{m}} \frac{V_{\mathrm{b}}^2}{2} \left[1 + \Upsilon \frac{C_{\text{in situ}}}{(1-k)k^2[1 + C_{\text{in situ}}(s-1)]}\right] \tag{43}$$

The first term on the right hand part of the equation indicates pressure drop occurring in the flow of single-phase mixture; the second and dimensionless term indicates pressure drop occurring due to the rolling movement of the capsules. The coefficient (Y) before the dimensionless term can be changed with reference to experimental findings. Calculations made with reference to the conclusions arrived at in the theoretical approach indicate that according to the experimental findings, the coefficient is 0.45 for capsule concentrations between 5% to and 30%. Comparison of the results calculated through Eq. (44) with the experimental findings revealed a match with an average deviation by 2.7% for $2.5 \times 10^4 < Re < 1.5 \times 10^5$ (min. deviation is 1.12%, max. deviation is 5.31%) (Fig. 10).

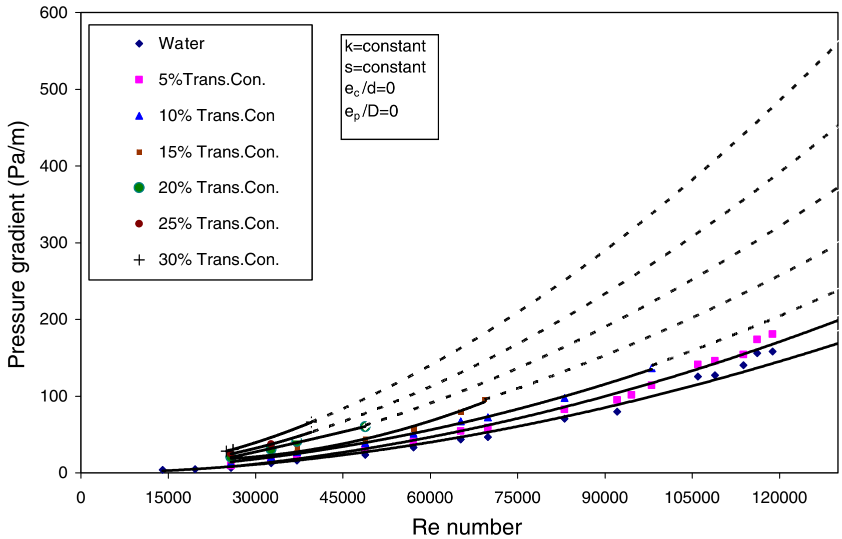


Fig. 11. $(\Delta P/L)_{\mathrm{m}} - V_{\mathrm{b}}$ variation calculated through Eq. (44) and experimental findings.

$$\left(\frac{\Delta P}{L}\right)_{\mathrm{m}} = \lambda_{\mathrm{p}}\frac{1}{D}\rho_{\mathrm{m}}\frac{V_{\mathrm{b}}^2}{2}\left[1+0.45\frac{C_{\text{in situ}}}{(1-k)k^2[1+C_{\text{in situ}}(s-1)]}\right] \tag{44}$$

The curves drawn with reference to Eq. (44) and to the experimental findings were based on dimensionless numbers (Fig. 11). The Froude number of the capsules was considered negligible; specific gravity, diameter ratio, and relative roughness of pipes and capsules were considered constants.

Fig. 6 is a graphic made of experimental values only. The curves given in this graphic were completed by using the mathematical model (Fig. 11). Fig. 11 is a graphic created by using Eq. (44) to calculate the pressure drops which can occur until the Re number where a jump can occur are reached. After reaching a certain velocity and concentration value, the flow regime will change. Therefore, no extrapolation should be made for the $Re > 1.5 \times 10^5$ values in Fig. 11 to calculate pressure drops.

9. Conclusion

Assuming the theoretical approach that "total pressure drop occurring in the flow of a mixture of capsule and water per unit length equals the pressure drop caused by the flow of single-phase mixture (homogenous flow) plus the pressure drop occurring due to the rolling movement of the spheres", a mathematical expression was developed to characterize the physical event of the capsule and water mixture flow. The first term on the right hand part of Eq. (44) indicates pressure drop occurring in the flow of single-phase mixture; the second and dimensionless term indicates pressure drop occurring due to the rolling movement of the capsules. Comparison of the results calculated through Eq. (44) with the experimental findings reveals a match with an average deviation by 2.7% for $2.5 \times 10^4 < Re < 1.5 \times 10^5$ (min. deviation is 1.12%, max. deviation is 5.31%).

Taking the Darcy–Weisbach equation as a basis, an empirical expression (experimental finding expression) was developed (Eq. (24)). Pressure drops calculated by using the empirical expression for capsule flows under different conditions (in the range $2.5 \times 10^4 < Re < 1.5 \times 10^5$) deviate from the actual values by 3.37% on average (min. deviation 1.70%, max. deviation 4.96%).

The capsules will move towards the axis of the pipe when the flow velocity is $Re > 1.5 \times 10^5$, pressure drops occurring in the flow with capsules will increase lesser and get close to pressure drops incurring under homogenous flow conditions. When the capsules move exactly on the axis (i.e. when they flow), no rolling moments will occur at all, so that pressure drops will equal to homogenous flow values. Therefore, it is inferred that the pressure drop curves drawn for unit length by means of Eq. (44) should not be extrapolated after the velocity value $Re > 1.5 \times 10^5$.

Acknowledgements

This research has been supported by various industrial corporations and Yildiz Technical University Scientific Projects Coordination Department. Project Number: 21-06-01-02.

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